MAT2705-01/02 13f Quiz 1 Print Name (Last, First) _____, _____|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $y' = x - y : gen\ sol : y(x) = C e^{-x} + x - 1$
   a) Verify that this y satisfies the given differential equation.
   b) Find the solution which satisfies the initial condition $y(0) = 10$.
Organize your work as though you were playing professor.

2. Choose appropriately named variables and write a differential equation that models the situation:
"A new widget hits the market with a predicted saturation number S (constant) of the number N of units sold. It is estimated that the time rate of change of this number is proportional to itself and its difference with the saturation number." [i.e., to the product of the two numbers].
Knowing that if N is below the saturation number, the sales should still increase, what is the sign of the constant of proportionality you introduced?

▶ solution

(understood: $N \geq 0$)

① a) $y = Ce^{-x} + x - 1$

$\frac{dy}{dx} = C(-e^{-x}) + 1 + 0$

$y' = x - y$

$-Ce^{-x} + 1 = x - (Ce^{-x} + x - 1)$

$= x - Ce^{-x} - x + 1$

$= -Ce^{-x} + 1$ ✓

b) $y(0) = 10 \leftrightarrow x = 0, y = 10$

$10 = Ce^{-0} + 0 - 1 = C - 1$

$C = 11 \rightarrow \boxed{y = 11e^{-x} + x - 1}$

$10 = C - 1 \not\rightarrow C = 9$!

THINK about equation manipulation!

$C = 10 + 1 = 11$ put in extra steps if necessary.

② $\frac{dN}{dt} \propto N(S-N)$ ← words to symbols

translation to equation →

$\boxed{\frac{dN}{dt} = kN(S-N)}$

if $N < S$, the $S - N > 0$

so $N(S-N) > 0$

if N is increasing, then $\frac{dN}{dt} > 0$

so $\boxed{k > 0}$

always justify your claims!

(if instead you chose $\frac{dN}{dt} = k(N)(N-S)$ then $k < 0$.)