MAT1505-01/02 21F Test 2 Print Name (Last, First) _____ , _____|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Maple may not substitute for any hand calculations unless explicitly stated, but use it to check each step if you want to be safe.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. a) Check that for the function $f(x) = \left(1 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$, $-1 \le x \le 1$, the following is true: $1 + f'(x)^2 = x^{-\frac{2}{3}}$.
b) Evaluate the arclength from $x = 0$ to $x = 1$, being careful to properly evaluate this improper integral.
c) Evaluate the arclength function $s(x)$ satisfying $s(0) = 0$, being careful to properly evaluate this improper integral.
d) Use it to confirm your result of part b).

2. A logistic distribution function has the expression: $p(x) = \dfrac{e^{3-x}}{\left(1 + e^{3-x}\right)^2}$, $-\infty < x < \infty$. Evaluate the probability that x takes a value between 2 and 4, exactly and numerically to 3 decimal places. [Hint: you can do this integral by hand!]
Optional only if you finish early: If you plot this for $x = 0..6$, this appears to be symmetric about the peak at $x = 3$. Use Maple to simplify the difference $f(3 + X) - f(3 - X)$ to show that its values equidistant from this peak agree.

3. Consider the probability distribution function $p(x) = 2\,x\,e^{-x^2}$, $0 \le x < \infty$, which peaks at $x_{max} = \dfrac{1}{\sqrt{2}} \approx 0.707$.
a) Verify that the total probability is 1, using limit notation appropriately.
b) Evaluate the mean value $\mu = \int_0^\infty x\,p(x)\,dx$ exactly using Maple, and numerically to 3 decimal places.
c) The median value is found by solving the equation $\int_0^m p(x)\,dx = \dfrac{1}{2}$. Solve this exactly by hand and numerically to 3 decimal places.
d) Make a rough sketch of $p(x)$ for $x = 0..3$, indicating by vertical lines from the horizontal axis up to the graph, the values x_{max}, μ, m.