

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYing expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). All integrals once their integrand is simplified may be evaluated with technology exactly or numerically if requested. Otherwise Maple can ONLY be used to check calculations, which should all be done by hand. Always justify limits of integration by mathematical reasoning. Maple may not substitute for any hand calculations unless explicitly stated, but use it to check each step if you want to be safe. Reading off a tech plot does not qualify as "deriving" a value of a variable.

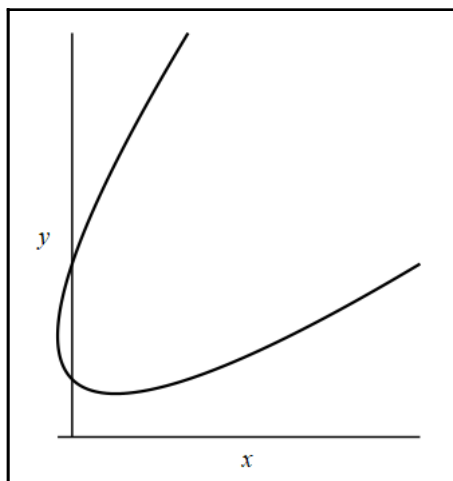
pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

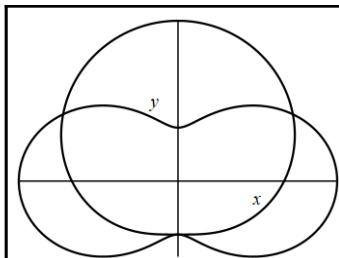
Signature: _____

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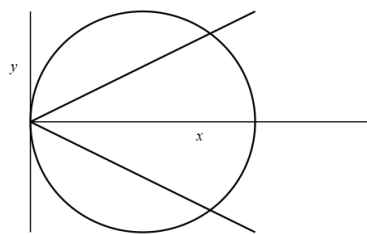
1. Consider the parametrized curve $x = t^2 - t$, $y = t^2 + t + 1$, $t = -2 \dots 2$. Plot this on your tech device.

- Derive the values of the vertical axis intercepts.
- Write down the equation of the tangent line at $t = -2$ and
- where does this line intersect the horizontal axis?
- Write down an integral representing the arclength of the portion of the curve in the second quadrant, simplify the integrand, and use Maple to evaluate it exactly and then numerically to 4 decimal places. Can you think of a way to compare this to see if the number makes sense? Explain.



2. Consider the two curves: $r = 2 + \sin(\theta)$, $r = 2 + \cos(2\theta)$, $\theta = 0 \dots 2\pi$. Plot this on your tech device to compare with your calculations.

- Find the exact intersection points in the upper half plane in polar and Cartesian coordinates. [Use the identity: $\cos(2\theta) = 1 - 2\sin^2(\theta)$ and the quadratic formula!]
- Set up an integral for the area of the region above/outside the lower double lobe curve and inside the other curve and simplify the integrand. Shade in this region in the diagram and draw in the half lines of constant theta which bound it, labeling them by your angular limits.
- Use Maple to evaluate the integral exactly and then numerically to 4 decimal places.
- The first curve looks almost like a circle of radius 2. What fraction of this comparison circle is the result of part c)? Is that reasonable?



3. a) Draw tangent lines from the two points of intersection of the oblique lines with the right side of the circle.

b) Consider the circle $r = 2 \cos(\theta)$ and the two lines $y = \pm \frac{1}{2} x$. Plot this on your tech device to compare with your calculations. Then find the exact polar and Cartesian coordinates of those two points of intersection.

c) Derive the equations for the two tangent lines at those intersection points. Where do they intersect?

d) Write down and simplify the integrand for the integral for the area of the wedge of the circle between these two lines. Then evaluate it using technology exactly and then numerically to 4 decimal places. Shade in this region of the diagram. Compare with the total area of the circle, does your result seem reasonable?

e) Write down and simplify the integrand for the integral for the arclength of the wedge of the circle between these two lines. Then evaluate it exactly and then numerically to 4 decimal places. [Does your fraction of the circular perimeter seem reasonable?]

Remember for a single parametrized curve, or a pair of polar coordinate curves, use PlotBuilder and copy and paste graphs together or:

```
> plot([t, t^2, t=0..1])
```

```
> plot([1, cos(theta)], theta=0..pi, color=[red, blue], coords=polar)
```

Remember: " " (space) or "*" for multiplication ALWAYS, $\pi \neq \text{pi}$, $e \neq e$. Context menu plots can be copied and pasted together. Gridlines and 1-1 option may be useful.

Do not put any work here!