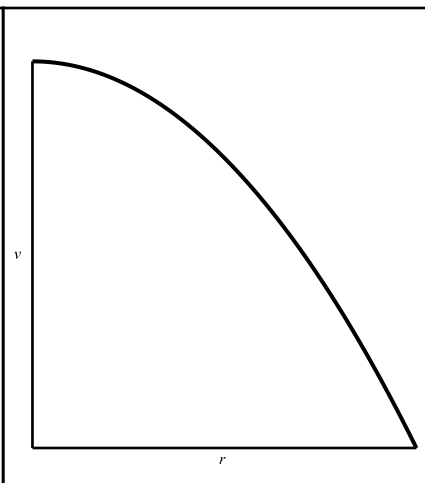


Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC).

Given the velocity profile $v(r) = \frac{P}{4 \eta \ell} (R^2 - r^2)$ for $0 \leq r \leq R$ for blood flow in a long cylindrical tube as a function of the radial distance r from the central axis of the tube of radius R , where $P > 0$ is the pressure (more pressure faster flow) $\eta > 0$ is the blood viscosity (thicker blood slower flow) and $\ell > 0$ is the length of the tube (idealized artery! longer tube, slower flow?). This is fast in the center and tapers off to zero at the outer edge. [See diagram.]

A more appropriate average velocity of the blood flow would be one weighted by the differential area $dA = 2 \pi r dr$ represented by the annular strip of thickness dr , to give more weight to the larger ring of blood flowing farther from the center, namely



$$\langle v \rangle = \frac{\int_0^R v(r) 2 \pi r dr}{\int_0^R 2 \pi r dr} = \frac{\int_0^R v(r) 2 \pi r dr}{\pi R^2}. \text{ [This line is all you need.]}$$

[One could roughly approximate the blood flow as a uniform velocity flow at this average velocity independent of radius. It would give the same total amount of blood passing any cross-section at any given moment. We will come back to this later in the course.]

- For the given velocity profile, introduce the change of variable $x = \frac{r}{R}$ or $r = x R$ into this integral expression to re-express it completely in terms of the new dimensionless variable x and re-express the coefficient of the resulting integral (don't evaluate the integral yet!) for the average velocity $\langle v \rangle$ in terms of the maximum velocity $v_{\max} = v(0)$.
- Now integrate by hand the resulting integral in the variable x .
- At what fraction of the radius R does the actual velocity equal this average velocity $\langle v \rangle$?
- Check your evaluation in part b) using Maple. Does it agree with your hand result as it should?

► solution