

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them unless otherwise indicated.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and **and scan** this test sheet as a cover first page in the PDF scan of your lined paper hand work all on separate sheets.

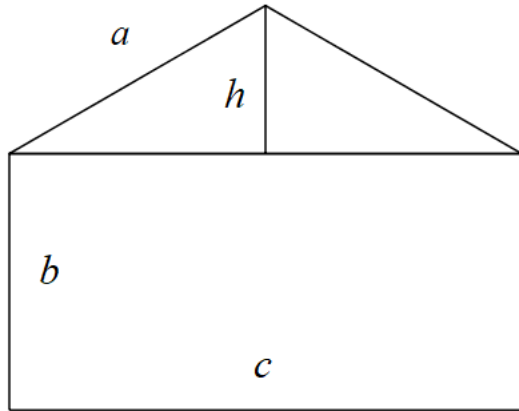
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date:

1. Classify the critical points of the function $f(x, y) = x^2 + y^4 + 2xy$ using the second derivative test (check both signs of the 2nd partials then the checking quantity!), giving their locations and function values. Do Maple 3d plots and countourplots confirm your numbers?
2. Find the maximum volume of a rectangular box that is inscribed in a sphere of radius r . Be sure you methodically draw a diagram, assign variables and their allowed values, describe the constraint and use it to eliminate one variable to get a function of the remaining two variables to maximize, and draw a 2d diagram of the allowed region for those two variables, labeling its boundary. Check that the single critical point you find (which on physical grounds has to be the unique maximum) is indeed a local maximum by the second derivative test. [You may use Maple to differentiate and simplify, but state the results.] State your final response as an English sentence.
3. a) The two legs of a right triangle are measured as 5 m and 12 m with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the (absolute value of the) maximum error in the calculated value of the length of the hypotenuse, to 4 decimal places.
b) For comparison with this number, how far above or below the calculated value of the hypotenuse using these measurements can the actual measurements yield for that hypotenuse length if they are within the error bars given, evaluated to 4 decimal places.
4. $x^2 + 2y^2 - 3z^2 = 3$, $P(2, -1, 1)$,
a) Write the simplified equation for the tangent plane to this surface at this point.
b) Write the simplest parametrized equations for the normal line to this surface at this point.
c) What angle (exactly and to one decimal place in degrees) does the normal line make with the horizontal?

5. Optional 10pts.



A pentagon is formed by placing an isosceles triangle on a rectangle, as shown in the figure. If the pentagon has fixed perimeter P , find the lengths of the sides of the pentagon that maximize the area of the pentagon.

a) Express the area in terms of a, b, c, h .

b) Express h in terms of a and c using the Pythagorean theorem.

c) Use the perimeter constraint to eliminate the variable b .

d) Write down the critical point equations for the area function $A(a, c)$.

e) Show that the first of these two equations can be solved for c as a function of a . [Note that this shows the right triangle is a 30-60-90 triangle.]

f) Use this to eliminate c from the second equation and solve it for a .

g) Evaluate the rest of the variables first in terms of a and then in terms of P and finally evaluate the resulting area in terms of P . Simplify your expressions.

► **solution**