

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them. Explain in as many words as possible everything you are doing!

**Back all double integrals with a fully labeled diagram like bob always requests** (consult previous tests, quizzes, handouts if in doubt).

## pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and **and scan** this test sheet as a cover first page in the PDF scan of your lined paper hand work all on separate sheets.

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Date:

Signature:

1. Given  $\int_C (-x^2) dx + x y dy$  over the closed curve which is the counterclockwise oriented closed curve in the first quadrant which is the boundary of the region  $R$  between the three curves

$$(x-2)^2 + y^2 = 4, x^2 + (y-1)^2 = 1, y = 0.$$

a) Make a diagram of this piecewise defined curve boundary and the region it encloses, finding the points of intersection and labeling them.

b) Evaluate the line integral directly using a polar coordinate parametrization of the two circles [first express  $r = r(\theta)$  then insert into  $r = \langle x(r, \theta), y(r, \theta) \rangle$ ] and use an obvious parametrization of the straight line segment. Be sure to state the simplified integrand before using technology to evaluate the iterated integral.

c) Use Green's Theorem to evaluate the equivalent double integral, again in polar coordinates. Be sure to state the simplified integrand before using technology to evaluate the iterated integral.

d) Check part c) using instead Cartesian coordinates to evaluate the double integral. You should get the same result.

e) **Optional. [Ignore this, it's for bob's amusement.]** If you add the counterclockwise line integrals of the vector field  $\frac{1}{2} \langle -y, x \rangle$  over the two circular curves in this boundary, you will get the area of the region, which you can check against either the Cartesian or polar double integral iteration for the area of this region  $R$ .

2. Given the parametrized curve  $r(t) = \langle \cos(t), \sin(t), \sin(2t) \rangle, t = 0 \dots 2\pi$ , and the vector field  $F(x, y, z) = \langle yz, zx, xy \rangle$ .

a) Set up and simplify the line integral of the vector field over this path, then use technology to evaluate it.

b) Set up and simplify the line integral of this vector field on the straight line segment from  $r(0)$  to  $r\left(\frac{\pi}{3}\right)$ .

c) Evaluate the divergence of this vector field.

3. Consider the vector field  $\vec{F}(x, y, z) = \left\langle 2x \ln(y), \frac{x^2}{y} + z^2, 2yz \right\rangle$  and the parametrized curve

$$r(t) = \langle t^2, t, t \rangle, t = 1 \dots e.$$

a) Set up and simplify the line integral of the vector field over this path, then use technology to evaluate it.

b) Show that  $\text{curl}(\vec{F}) = 0$  (hand derivatives in the formula) wherever this vector field is well defined and therefore admits a local potential function.

c) Solve the equations which determine that potential  $f(x, y, z)$ .

d) Use it to evaluate the line integral between the endpoints of the above parametrized curve to check your work.