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Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them.

Given three points $P_{1}(1,1), P_{2}(-1,6), P_{3}(5,0)$ in the plane:
a) Evaluate the coordinates of the midpoint $M$ of the line segment $P_{2} P_{3}$. Let $\overrightarrow{\boldsymbol{b}}=\overrightarrow{P_{1} M}$. On the reverse side of this sheet (left grid), BEFORE doing this problem, draw in the three points and their position vectors, labeling each point and vector ( $\overrightarrow{O P}_{1}$ or $\vec{r}_{1}$ etc), and draw in the triangle that the points determine, labeling the two sides $\overrightarrow{P_{1} P_{2}}$ and $\overrightarrow{P_{1} P_{3}}$ by appropriate symbols for their difference vectors and put in arrow heads to indicate which direction your vector symbol for each side refers to, and draw in the vector $\overrightarrow{\boldsymbol{b}}$ from the point $P_{1}$, indicating the point $M$ at its tip.
b) Next drop a perpendicular from $M$ to the side $P_{1} P_{2}$ and complete this to the projection rectangle appropriate for $\rightarrow$ projecting $\overrightarrow{\boldsymbol{b}}$ along that side of the triangle and orthogonal to it. Identify the vector projections $\overrightarrow{\boldsymbol{b}}_{\|} \overrightarrow{\boldsymbol{b}}_{\perp}$ in the diagram as arrows with initial points at $P_{1}$ labeled by their symbols.
c) Give a very rough estimate of the angle that $\overrightarrow{\boldsymbol{b}}$ makes with $\overrightarrow{P_{1} P_{2}}$, then evaluate it exactly (no decimals) and numerically to the nearest tenth of a degree (don't change your initial estimate!). Does your result seem consistent with estimate? Explain. [Not even I can estimate an angle better than 10 degrees without a protractor, but it should be in the same ballpark, so to speak.]
e) Estimate roughly the numerical components of the two vector projections using the grid and interpolation. f) Now using appropriate notation, step by step (show every step starting from the initial vector components), evaluate the vector components of the parallel and perpendicular projections that you have drawn, exactly and then to 2 decimal place accuracy.
g) How do the numerical evaluations of your exact vectors compare to your graphical estimates? Do they seem consistent? Explain.

## solution

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P_{1}(1,1), P_{2}(-1,6), P_{3}(5,0)
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