MAT2705-04/05 22S Test 2   Print Name (Last, First) _________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses without showing details (identify technology).** Otherwise only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested. [Make sure you check every solution using Maple!]

pledge **[sign and date the pledge at the end of your exam]**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. Let $A = \langle v_1 | v_2 | v_3 | v_4 \rangle = \begin{bmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \end{bmatrix}$ define 4 vectors in $\Re^3$.

a) Write out the matrix equation $A\,x = 0$ (without multiplying out). Solve this system of equations by using Maple to do the appropriate augmented matrix row reduction, then identifying free and leading variables and writing out the reduced equations, solving them and parametrizing the family of solutions. [If you don't get integer coefficients, you mis-entered a matrix entry.]

b) Express the solution as a linear combination of column matrices which form a basis of the solution space.

c) Use this basis to express the independent linear relationships among these 4 vectors.

2. a) Use an inverse matrix to express the vector $\langle -7, 7, 11 \rangle$ as a linear combination of the vectors $\langle 1, 2, 1 \rangle$, $\langle -4, -1, 2 \rangle$, $\langle -3, 1, 4 \rangle$. State first the matrix equation you are solving to accomplish this, and show explicitly the matrix multiplication steps in solving it. [The result consists of integers if you entered numbers correctly.]

b) Once you have accomplished this, explicitly evaluate this linear combination to confirm that it does the job.

[put all other work and responses on separate sheets except for the page 2 graph work]

3. a) On the grid below, **draw in** arrows representing the new basis vectors $\vec{b}_1 = \langle 2, 1 \rangle$ and $\vec{b}_2 = \langle -2, 3 \rangle$ and $\vec{v}_3 = \langle 0, -8 \rangle$ and **label** them by their symbols. **Extend** the basis vectors $\left\{ \vec{b}_1, \vec{b}_2 \right\}$ to the corresponding coordinate axes for $\left(y_1, y_2 \right)$ and **mark** the positive direction with an arrow head and the axis label. Mark off tickmarks on these axes for integer values of the new coordinates.Then **draw in** the parallelogram with edges parallel to the new axes for which $\vec{v}_3$ is the main diagonal and shade it in in pencil lightly. Read off the coordinates $\left(y_1, y_2 \right)$ of $\vec{v}_3$ with respect to these two vectors (write them down) and **express** $\vec{v}_3$ as a linear combination of these vectors; **put this equation** at the tip of this vector: $\vec{v}_3 = (\ ...)\ \vec{\boldsymbol{b}}_1 + (\ ...)\vec{\boldsymbol{b}}_2$

b) Now use the inverse basis changing matrix to express $\vec{v}_3$ as a linear combination of the two two vectors (show all steps in this process, including the details of matrix multiplication and arithmetic), box it and then check your linear combination by expanding it out. Does your matrix result agree with your graphical result in part a)?

c) **Draw in** the arrow representing the vector $\vec{\boldsymbol{v}}_4$ whose new coordinates are $\left(y_1, y_2 \right) = (4, 2)$ and **label** the tip of $\vec{\boldsymbol{v}}_4$ by its symbol. Draw in the projection parallelogram associated with the new coordinates and lightly shade it in in pencil. Determine its old coordinates $\left(x_1, x_2 \right)$ graphically. Then evaluate them using a linear combination or matrix product.