MAT2705-04/05 23s Quiz 5 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **Only use technology to CHECK hand calculations, not substitute for them, unless specifically requested.**

1. For the homogeneous linear system of equations $A\,x=0$:

$$\begin{aligned} x_1 + 3\,x_2 + 8\,x_3 - 1\,x_4 &= 0\\ 2\,x_1 + x_2 + x_3 + 3\,x_4 &= 0\\ x_1 + 4\,x_2 + 11\,x_3 - 2\,x_4 &= 0 \end{aligned}$$

a) Write down the augmented matrix and use technology to fully reduce it.
b) Label the appropriate columns by L or F and identify explicitly the leading (L) and free (F) variables from the form of the reduced matrix.
c) Write down the all reduced equations and solve them indicating your steps to express your solution as a column matrix equation for the column matrix variable x.
d) Rewrite your solution as an arbitrary linear combination of fixed entry matrices whose coefficients are the arbitrary parameters: $x = t_1\,u_1 + ...$
e) Show by matrix multiplication with the coefficient matrix that each of your fixed entry matrices to d) are the zero column matrix. Show your arithmetic explicitly in evaluating those matrix products.

2. Consider the system

$$\begin{aligned} 8\,x_1 + 15\,x_2 &= 7\\ 5\,x_1 + 10\,x_2 &= 5 \end{aligned}$$

a) Write down explicitly the matrix form of this linear system.
b) Write down the coefficient matrix A and evaluate its inverse from the memorized formula for 2x2 inverses (check with technology).
c) Use it to solve the system for the column matrix x of solutions.

▶ **solution**

① a)
$$\begin{bmatrix} 1 & 3 & 8 & -1 & 0\\ 2 & 1 & 1 & 3 & 0\\ 1 & 4 & 11 & -2 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{array}{c} \begin{array}{ccccc} L & L & F & F & \\ x_1 & x_2 & x_3 & x_4 & \end{array}\\ \begin{bmatrix} 1 & 0 & -1 & 2 & 0\\ 0 & 1 & 3 & -1 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b) $\longrightarrow$ leading : x_1, x_2
free : x_3, x_4

$\downarrow$

c) $x_3 = t_1$
$x_4 = t_2$

c)
$$\begin{aligned} x_1 \quad - x_3 + 2x_4 &= 0 &\rightarrow\ x_1 &= x_3 - 2x_4 = t_1 - 2t_2\\ x_2 + 3x_3 - x_4 &= 0 &\rightarrow\ x_2 &= -3x_3 + x_4 = -3t_1 + t_2\\ 0 &= 0 \end{aligned}$$

$$\begin{bmatrix} x_1\\ x_2\\ x_3\\ x_4 \end{bmatrix} = \begin{bmatrix} t_1 - 2t_2\\ -3t_1 + t_2\\ t_1\\ t_2 \end{bmatrix} \overset{d)}{=} \begin{bmatrix} 1t_1 - 2t_2\\ -3t_1 + 1t_2\\ 1t_1 + 0t_2\\ 0t_1 + 1t_2 \end{bmatrix} = t_1 \begin{bmatrix} 1\\ -3\\ 1\\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -2\\ 1\\ 0\\ 1 \end{bmatrix}$$

e)
$$\begin{bmatrix} 1 & 3 & 8 & -1\\ 2 & 1 & 1 & 3\\ 1 & 4 & 11 & -2 \end{bmatrix} \begin{bmatrix} 1\\ -3\\ 1\\ 0 \end{bmatrix} = \begin{bmatrix} 1 - 9 + 8 + 0\\ 2 - 3 + 1 + 0\\ 1 - 12 + 11 + 0 \end{bmatrix} = \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 1 & 3 & 8 & -1\\ 2 & 1 & 1 & 3\\ 1 & 4 & 11 & -2 \end{bmatrix} \begin{bmatrix} -2\\ 1\\ 0\\ 1 \end{bmatrix} = \begin{bmatrix} -2 + 3 + 0 - 1\\ -4 + 1 + 0 + 3\\ -2 + 4 + 0 - 2 \end{bmatrix} = \begin{bmatrix} 0\\ 0\\ 0 \end{bmatrix} \checkmark$$

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(2) $8x_1 + 15x_2 = 7$
$5x_1 + 10x_2 = 5$

a) $\begin{bmatrix} 8 & 15 \\ 5 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$

b) $A = \begin{bmatrix} 8 & 15 \\ 5 & 10 \end{bmatrix} \quad A^{-1} = \frac{1}{\underbrace{8\cdot 10 - 5\cdot 15}_{= 80-75=5}} \begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix} = \boxed{\frac{1}{5}\begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix}}$

c) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A^{-1} \begin{bmatrix} 7 \\ 5 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 10(7) - 15(5) \\ -5(7) + 8(5) \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 70-75 \\ -35+40 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -5 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

so $\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$